The Absence of a Size Effect Relevant to the Cost of Equity

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In this paper, I evaluate whether there is a size effect that is relevant to the cost of equity. I first analyze what model investors use to determine the required rate of return on their investment and find investors prefer the Capital Asset Pricing Model (CAPM) over other models, even those that include a size proxy. I also show that over the period 1981 to 2016, small stocks underperformed large stocks, which is inconsistent with the existence of a size effect. Finally, I conclude that size effect studies have not been able to surmount the criticisms that the size effect lacks a theoretical basis and that the results of size effect studies are susceptible to data mining criticisms. Given these results, practitioners should reconsider the standard practice of augmenting their cost of equity with a size premium.

Introduction

The cost of equity capital is a critical component when valuing a firm. It is used as the rate to discount equity cash flows, or it is a component of the weighted-average cost of capital used to discount firm cash flows. The Capital Asset Pricing Model (CAPM) is typically used when calculating the cost of equity. Standard finance textbooks warn against adding “arbitrary fudge factors” to the discount rate, but practitioners often augment or modify their cost of equity with a size premium reported in publications by Morningstar/Ibbotson and Duff & Phelps. The size premium is thought of as compensation for the outperformance by small stocks relative to large stocks on a risk-adjusted basis. There are studies that document such outperformance, but there are also studies that show the size effect does not exist, the size effect vanished in the 1980s, and the methods, inputs, and/or assumptions in the size effect studies are flawed.

As it pertains to valuation, a potential source of confusion among the size effect studies is the conflating of the potential impact of size on expected cash flows with the potential impact of size on the cost of equity. In this paper, I attempt to clarify this confusion by investigating whether the size effect has an impact on the cost of equity. The results of my analyses suggest that there is an absence of a size effect that is relevant to the cost of equity. Consequently, practitioners may have to reconsider the standard practice of augmenting their cost of equity with a size premium. This also implies that any

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3 This definition of a size premium is consistent with the definition in Banz (1981), the seminal paper on the size effect: Rolf Banz, “The Relationship Between Return and Market Value of Common Stocks,” Journal of Financial Economics 9 (1981):3–18. However, some size effect studies advocate for a different measure of size, such as book value of assets, sales, and number of employees. See, for example, Roger Grabowski, “The Size Effect—It Is Still Relevant,” Business Valuation Review 35 (2016):62–71. However, other studies have shown that some of these alternative measures of size are not related to returns. See, for example, Jonathan Berk, An Empirical Re-Examination of the Relation Between Firm Size and Return. Working Paper (Seattle: University of Washington, 1996).
4 Banz (1981); Eugene Fama and Kenneth French, “The Cross-Section of Expected Stock Returns,” Journal of Finance 47(2) (1992):427–465. Given this lack of consensus about whether these alternative measures of size are related to returns, I focus on a metric for which there appears to be consensus in the literature (i.e., market capitalization) as the measure of size in this paper.
adjustments related to a size effect, if necessary, should likely be made to the expected cash flows.6

In this paper, I first investigate how investors estimate their required rate of return when making investments. Note that there has been a substantial amount of empirical evidence that shows the poor performance of the CAPM. A prominent strand is the size effect, which began with Banz (1981) and continued on through the numerous size effect studies that have been published since that time.7 Therefore, it would be instructive to know whether investors, who put their money on the line, use the CAPM or models that incorporate the size effect when determining their required rate of return. My review of the evidence finds that investors prefer the CAPM over models that include a size proxy, such as the Fama-French model. This could imply that most investors fall into one of two types. The first type includes investors that do not believe a size effect exists. The second type represents investors that believe a size effect exists, but they believe the impact should not be accounted for in the cost of equity (i.e., any impact attributable to or proxied by the size effect should be accounted for in the expected cash flows). Consequently, practitioners that augment their cost of equity with a size premium appear to be using a cost of equity that is inconsistent with most investors’ actions.

Next, I analyze whether small capitalization stocks have outperformed large capitalization stocks after Banz (1981) was published. Many size effect studies include pre-1981 data, which have been demonstrated to bias the results towards finding a size effect.8 In addition, there have been significant developments affecting small firms after the publication by Banz (1981), such as the proliferation of other size effect studies and the founding of small-firm mutual funds. Because of this structural shift, using data prior to 1981 is less relevant to understanding whether a size effect exists today. Consequently, I use actual returns from 1981 to 2016 in my analysis to capture all developments since this structural shift. If a size effect exists, we would expect small stocks to outperform large stocks. However, I find the opposite. From 1981 to 2016, small stocks actually underperformed large stocks. This result is inconsistent with there being a size effect in general, let alone a size effect that is relevant to the cost of equity.

Finally, I evaluate whether there is a common set of criticisms that affect the size effect studies that such studies have not been able to overcome. After my review of the literature, I find that the size effect articles likely suffer from at least one of two major criticisms. The first major criticism is that the size effect lacks a theoretical basis. Without a theoretical basis, we cannot understand why size should matter. The second major criticism is that the results in size effect studies are susceptible to data mining criticisms. The data mining criticism can stem from the lack of theoretical basis, but it can also be an independent issue, as small changes to the assumptions and/or inputs used may make the findings of many size effect studies go away. Recall that the scientific method puts the burden of proof on the party that has claimed to have observed an anomaly, i.e., those finding that a size effect exists.9 However, the consistency of these two criticisms across the size effect studies since 1981 suggests that the size effect studies may have not met the required burden of proof.10

**Investors Do Not Appear to Demand Compensation for Size**

Whether investors demand compensation for size goes to the heart of whether there is a size effect that is relevant when estimating the cost of equity. There is extensive literature that shows the CAPM does not perform well empirically and that additional risk factors may need to be added to models when estimating the discount rate. However, given the proliferation of size effect studies in the academic and practitioner literature since the 1980s, it would be insightful to understand whether these size effect studies have had an impact on investors when investors determine their required rate of return. In this section, I analyze the evidence indicating whether investors’ actions when setting their discount rate is consistent with the CAPM or models that incorporate additional risk factors, such as size. If investors, who put their money on the line, do not demand a compensation for size in their required rate of return, we would expect to observe that investors prefer to use the CAPM when estimating their required rate of return. Otherwise, we would observe that investors prefer to use a multifactor model that includes a size proxy, such as the Fama-French model.

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6 Note that there are some studies that find a size effect when one accounts for changes to the cash flows. See Kewei Hou and Mathijs van Dijk, “Profitability Shocks and the Size Effect in the Cross-Section of Expected Stock Returns,” paper presented at the 2011 European Finance Association 38th Annual Meeting (Stockholm, Sweden: European Finance Association, 2011).


8 For example, see Black (1993) (footnote 5).


10 Interestingly, although there is no evidence that the burden of proof has shifted, valuation practitioners that do not add a size premium often find themselves in the position of having to defend their choice of not adding a size premium. See, for example, Aswath Damodaran, “The Small Cap Premium: Where Is the Beef?” Musings on Markets Blog (April 11, 2015), accessed at http://aswathdamodaran.blogspot.com/2015/04/the-small-cap-premium-fact-fiction-and.html, May 18, 2017.
The following three relevant studies provide helpful insights to answer my query. First, a 2017 study by Berk and van Binsbergen used mutual fund flows from a sample of 4,275 mutual funds covering the period January 1977 to March 2011. This study focused on the behavior of mutual fund investors, which covers a great majority of households with an annual income of over $100,000. The authors found the CAPM was the model that was most consistent with how mutual fund investors set their required rate of return. Moreover, the authors also found that the additional factors in the Fama-French model, which includes a size proxy, did not add explanatory power.

Second, a 2015 study by Pinto et al. surveyed professional equity analysts that are members of the CFA Institute. The equity analysts in their sample spent a majority of their time evaluating individual securities for purposes of making investment recommendations or portfolio decisions. The authors found that 68% of the 1,436 equity analysts that responded to their survey used the CAPM. By contrast, the authors found that the Fama-French model was used by less than 5% of respondents.

Last, a 2002 study by Graham and Harvey surveyed Fortune 500 chief financial officers (CFOs) and financial officers from 4,440 firms who were members of the Financial Executives Institute. Among other things, the survey investigated how the respondents made capital budgeting (i.e., investment) decisions. Based on the survey responses, the authors found that over 70% of their survey respondents always or almost always used the CAPM. Interestingly, a multifactor CAPM only ranked third and was used less frequently than the simplistic approach of using the firm’s average stock return as the cost of equity.

The wide cross section of investor types, with varying degrees of financial sophistication (i.e., from individual investors to professional equity analysts and financial executives), and the date range covered in these studies add to the robustness of the results. These results are also consistent with academic research showing that, despite the empirical evidence against the CAPM, the CAPM may still provide a reasonable estimate of a project’s cost of capital.

These results imply that most investors fall into one of two types: (1) investors that do not believe a size effect exists and, therefore, do not demand compensation for it, or (2) investors that believe a size effect exists, but believe the adjustment for the size effect is not made in the cost of equity. For example, any necessary adjustment could be done in the expected cash flows. Consequently, practitioners that augment their cost of equity with a size premium expose themselves to using a cost of equity that is inconsistent with how most investors set their required rate of return.

Small Stocks Do Not Outperform Large Stocks

The basic premise of the size effect is that small stocks underperform large stocks on a risk-adjusted basis. In this section, I test this premise by first running a simple test that compares how small capitalization stocks perform relative to large capitalization stocks on a raw return basis. I used value-weighted size-based decile returns obtained from Kenneth French’s Data Library. I used the smallest size-based decile as a proxy for small stocks and the largest size-based decile as a proxy for large stocks. I performed my comparison over the period 1981 to 2016, which is the period after the publication of the first size effect studies and the founding of small-cap mutual funds. My analysis shows that $100 invested in small stocks would have grown to $3,221 over the period, while the same $100 invested in large stocks would have grown to $3,774. In other words, small stocks underperformed large stocks by 12% over the period 1981 to 2016. Since small stocks already underperformed large stocks on a raw return basis, it follows that small stocks would underperform large stocks even more on a risk-adjusted basis, because small stocks are assumed to have higher risk or betas relative to large stocks. This result is inconsistent with the existence of a size effect, let alone adjusting the cost of equity with a size premium.

My finding is consistent with many finance textbooks that report the size effect vanishing in the 1980s. For example, one textbook explains: “The small-firm effect completely disappeared in 1980; you can date this as the publication of the first small-firm effect papers or the

12 In unreported results, using data from Kenneth French’s Data Library, I find that over the 1981 to 2016 period, the CAPM alone does as well as a two-factor model that consists of the excess market return and size proxy in explaining average portfolio returns. This also suggests that adding a size factor does not add explanatory power.
16 In unreported results, I find that there is no reliable relation between size and betas. Using the same data from Kenneth French’s Data Library, I find the beta of the largest size-based decile is indeed smaller than the beta of the smallest size-based decile. However, the beta of the portfolios in between these two extreme portfolios are in-line or, in most cases, higher than the beta of the smallest size-based decile. Hence, we do not observe a monotonic increase in betas as size decreases.
founding of small-firm mutual funds made diversified portfolios of small stocks available to average investors.”17 Another textbook notes: “Since the mid-1980s, however, there has been no size premium after adjusting for market risk.”18 One more textbook finds that “the abnormal performance of the DFA US 9-10 Small Company Portfolio, which closely mimics the strategy described by Banz (1981) . . . is insignificantly different from 1.0 in the period January 1982–May 2002 . . . Thus, it seems that the small-firm anomaly has disappeared since the initial publication of the papers that discovered it.”19 Note, however, that there are some studies that show the existence of a size effect for sample periods beginning in the 1980s, if one accounts for cash flow shocks.20 This result suggests that, to the extent there is a size effect, the adjustment should be made to the expected cash flows and not to the cost of equity.

Some recent studies, however, have used a different sample period to analyze whether a size effect exists. For example, a 2016 study by Grabowski used the period from 1990 to 2014 and found a size effect.21 Consistent with the author’s findings, my method above shows that small capitalization stocks outperformed large capitalization stocks over the period 1990 to 2016. However, I am not aware of a justification in the article by Grabowski for using 1990 as the start date. If I choose an equally arbitrary start date of 2000 or 2005 instead of 1990, my analysis would find that small stocks once again underperformed large stocks. This result could imply that finding a size effect when using a sample period beginning in 1990 may not be robust. However, these results from alternative start dates are not as meaningful because, in my opinion, there is no credible justification for starting the sample period on those dates, as these later start dates would not have captured all the relevant data after the structural shift related to the size effect that began in the early 1980s.22

Size Effect Studies Consistently Suffer from Two Criticisms

Many articles have criticized the size effect studies. For example, some articles criticize the size effect studies for using an improper risk measure or exhibiting an errors-in-variables bias.23 However, many of these issues are specific to a particular study and not reflective of a systematic issue. As such, I examined size effect articles in search of the criticisms that appear to be more common across the size effect literature, as existence of such long-running and unresolved issues provides evidence of criticisms that appear harder to surmount. I find that size effect studies suffer from at least one of two major criticisms: the lack of a theoretical basis for a size effect, and the susceptibility of the results to data mining. For ease of exposition, I elaborate on these two criticisms by using Banz (1981) and Fama-French (1992) as my primary examples, as these are two of the most commonly cited articles used to support the existence of a size effect.24 Ultimately, without overcoming these two criticisms, it does not appear that the size effect studies can surmount the burden of proof required to establish an impact of the size effect on the cost of equity.

Lack of a theoretical basis

The first major criticism of size effect studies is the lack of a theoretical basis for finding a size effect. This means that the articles do not give us a reason why we should expect a size effect. For example, Banz (1981) concludes by admitting, “[t]here is no theoretical foundation for such an effect.” As for Fama-French (1992), Fischer Black observes: “Fama and French also give no reasons for a relation between size and expected return.”25 There have been articles that have attempted to come up with a theoretical basis for the size effect, but, to the best of my knowledge, these theories have yet to obtain empirical confirmation beyond the simulated results provided.26

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17 Cochrane (2005).
18 Ang (2014).
20 Hou and van Dijk (2011).
21 Grabowski (2016).
22 In addition, one would also need a sufficiently long period to analyze the size effect. Although there is no bright line test to determine the length of the sample period, Fama and French suggest a period of at least thirty-five years is necessary to be confident with the results. See Eugene Fama and Kenneth French, “Q&A: Small Stocks for the Long Run,” FamaFrench Forum (January 23, 2012), accessed at https://famafrench.dimension.com/questions-answers/qa-small-stocks-for-the-long-run.aspx, May 21, 2017. Starting the sample period in 1981 satisfies this thirty-five-plus-year threshold.
24 For a recent survey of the size effect literature, see Mathijs van Dijk, “Is Size Dead? A Review of the Size Effect in Equity Returns,” Journal of Banking and Finance 35 (2011):3263–3274. The author reaches similar findings regarding the two criticisms of size effect studies I discuss here. In particular, the author states: “In short, I find that the empirical evidence for the size effect is consistent at first sight, but fragile at closer inspection. I believe that more empirical research is needed to establish the validity of the size effect in both US and international stock returns. . . . I hesitate to recommend the application of an empirically inspired asset pricing model while there is ambiguity about the robustness and the causes of the size effect it incorporates.”
Numerous studies have claimed that size may be a proxy for some other factor. For example, Banz (1981) asserts that size may be a proxy for one or more unknown factors that are correlated with size. This assertion has been disputed by some studies, but some articles have claimed to identify specific factors. If such a factor can be identified, I believe that the first-best option is to adjust the expected cash flows rather than the discount rate for such effects. For example, the most commonly associated factor with size is illiquidity. Some researchers have defined liquidity as “the speed at which a large quantity of a security can be traded with a minimal impact on the price and with the lowest transaction costs.” However, when costs and constraints to trading due to illiquidity are present, the price of a security could deviate temporarily from the value of the security. This is particularly true when the large block trade has no signaling effect about the value of the firm, but the trade is brought about by an investor-specific need (e.g., the investor needs the funds to buy a yacht). Consistent with this, studies have modeled the impact of illiquidity as a temporary effect that is uncorrelated with fundamental value. To the extent that the block trade is driven by value-relevant information, I believe it would be more appropriate to model the value impact of this effect by adjusting the expected cash flows when it occurs rather than adding a premium to the discount rate.

**Data mining**

The second major criticism of size effect studies is that the results are susceptible to data mining criticisms. This does not mean that size effect studies are all a product of data mining, but that it is difficult to objectively distinguish a legitimate method used to find the size effect from that of data mining.

One potential indicator of data mining is that the effect does not stem from theory. Without a theory, the results are simply an artifact of the data used. Hence, given the above discussion on the lack of a theoretical basis plaguing size effect studies, the results of size effect studies are especially susceptible to the data mining criticism.

However, even if a credible theory does emerge, the volume of articles that question the robustness of many size effect studies still makes data mining a primary concern. For example, the results of many size effect studies are not robust to small changes in its inputs and assumptions. Let us take the choice of sample period as an example. Even Banz (1981) admits that the size effect is not very stable through time, and an analysis of the ten-year subperiods in his sample of New York Stock Exchange firms from 1926 to 1975 shows substantial differences in the magnitude of the size factor coefficient. As for Fama-French (1992), they use a sample period that overlaps with Banz (1981) but their results for the size effect go away when only the post-Banz data set is used. In fact, a 2012 study by Fama and French confirmed the lack of a size effect when using a more recent sample period. To overcome the criticism of potential data mining, authors of size effect studies must take great care in providing a justification for their sample period and ensure that their results are robust to equally plausible alternative sample periods.

Another sign of potential data mining is that studies have found that the size effect is only observed under very specific situations. For illustrative purposes, I discuss two common examples here. First, the size effect studies initially grouped firms into deciles or ten size-based portfolios. As it became harder to find a size effect when grouping stocks into deciles, more recent size effect studies have begun grouping stocks into twenty-five size-based portfolios. However, finding results by cutting and slicing the data to find patterns could be an indication of data mining. At the very least, such a change in bucketing methodology makes it difficult to distinguish a legitimate result from that of data mining.

The second example is that a size effect is found only in January, during which half of the effect is observed

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29 Grabowski (2016).
36 For example, see Grabowski (2016).
37 Brealey, Myers, and Allen (2011).
during the first five trading days of January.\(^3\) The most common rationale provided for this so-called January effect is that investors sell stocks with capital losses at the end of the tax year, which is December in most cases, to offset taxable income during that same tax year. As it relates to small stocks, the argument is that small stocks are likely candidates for tax-loss selling because their high volatility would lead to higher probabilities of larger capital losses.\(^4\) The January effect is then observed when the small stocks’ price rebounds back to its fundamental value in the beginning of the new tax year, when the selling pressure has been alleviated. However, the January effect has no impact on the value of the firm, and the effect observed is a temporary price effect due to factors unrelated to the firm’s fundamental value (i.e., the investors’ tax strategy). Therefore, to the extent that the January effect exists and is a function of size, the effect is only temporary, and we should not expect it to have an effect when determining the firm’s cost of equity.

Finally, another sign of potential data mining is when results are inconsistent with expectations. For example, if the size effect held up consistently, we would expect size premiums to monotonically increase as you go from a bucket of larger stocks to a bucket of smaller stocks. However, as an example, the size premium in excess of CAPM reported in the Ibbotson Stocks, Bonds, Bills, and Inflation (SBBI) Yearbooks for 2002, 2006, and 2015 violates this expectation.\(^4\) Note that these inconsistencies persist despite the fact that data used by Ibbotson includes returns for several decades prior to 1981.

\(^{41}\) Another potential criticism of the size premium in excess of CAPM by Ibbotson is that the reported size premium is estimated using a methodology that is inconsistent with how valuation practitioners estimate their CAPM cost of equity. See Clifford Ang, “Why We Shouldn’t Add a Size Premium to the CAPM Cost of Equity,” NACVA QuickRead (February 15, 2017), accessed at http://quickreadbuzz.com/2017/02/15/shouldnt-add-size-premium-capm-cost-equity/, December 12, 2017.

**Conclusion**

My review of the evidence and analysis suggest that there may be no size effect that is relevant to the cost of equity. First, I find that, even after the publication of numerous size effect studies, investors prefer to use the CAPM when setting their required rate of return when they make investments. In particular, they choose the CAPM over models that include a size proxy, such as the Fama-French model. Second, I show that over the post-Banz (1981) period (1981 to 2016), small capitalization stocks underperformed large capitalization stocks. This result appears to question whether a size effect exists at all, let alone whether there is a size effect that impacts the cost of equity. Finally, I determined that size effect studies have had difficulty surmounting two major criticisms: lack of a theoretical basis for a size effect, and susceptibility of results to data mining criticisms. Given all of these factors, practitioners should reconsider the standard practice of augmenting their cost of equity with a size premium. To the extent that there is a direct or indirect impact of size on the value of the firm, in my opinion, the first-best option is to make the relevant adjustment to the expected cash flows.

Note that there is a considerable amount of evidence that demonstrates the CAPM performs poorly in empirical tests. There have been criticisms of such studies, but a possible rationale for such poor performance may be missing risk factors that have yet to be identified. The results I present herein suggest that size may not be one of those missing risk factors. Therefore, finding these risk factors that replace size could be a fruitful area for future research.